4-vectors and energy-momentum conservation

Exercise 1 Lecture: from $\widetilde{u}=\gamma(c,\vec{v})$ show that $\widetilde{w}=(\gamma^4\vec{a}.\vec{\beta}\ ,\ \gamma^4(\vec{a}.\vec{\beta})\vec{\beta}+\gamma^2\vec{a})$.

Exercise 2 A rocket is in rectilinear motion with a constant proper acceleration a_p.

Initial conditions in the inertial frame *R*: at *t=0*, *v=0*, *x=0* and $\vec{a}(t=0)=a_p\,\vec{i}$.

1) Give $\widetilde{w}_{R_p}$ in the proper inertial coinciding frame of reference.
2) Apply the Lorentz boost to obtain the four-acceleration $\widetilde{w}_R$.
3) Demonstrate that $a(t)=a_p/\gamma^3$.
4) Find the expressions $v(t)=\dfrac{a_p t/c}{\sqrt{1+(a_p t/c)^2}}$ and $\gamma(t)=\sqrt{1+a_p^2t^2/c^2}$.
5) Find the hyperbola equation $c^2t^2-(x+c^2/a_p)^2=-c^4/a_p^2$.
6) From $\tau=\int\gamma\,dt$ find for the proper time $\tau=argsh(a_p t/c)$.

Exercise 3 In the galactic inertial frame a rocket speed at the constant velocity 70% *c*. The rocket shoot a proton with an angle of 45° with respect the forward direction and a speed of 90% *c* in the rocket frame. Find the components of the proton velocity in the galactic frame using two methods:

1) Use the 4-velocity $\widetilde{u}$ and is covariance.

2) Demonstrate from the Special Lorentz Transform, and use the following formulas between the velocity expressed in *R*, $\vec{v}=(v_x,v_y,v_z)$, and in *R'*, $\vec{v}\,'=(v'_x,v'_y,v'_z)$:

$$\vec{v}\,'=\left(\frac{v_x-u}{1-\frac{u\,v_x}{c^2}},\frac{v_y}{\gamma\left(1-\frac{u\,v_x}{c^2}\right)},\frac{v_z}{\gamma\left(1-\frac{u\,v_x}{c^2}\right)}\right)\quad\text{with}\quad\vec{v}_{R'/R}=u\,\vec{i}$$

Exercise 4 2-Body decay

In the inertial reference frame *R* of the laboratory, a meson K^+ at rest decays into a meson π^+ and a meson π^0. The masses at rest are $m_{K^+}=987\,m_e$, $m_{\pi^+}=273\,m_e$, $m_{\pi^0}=264\,m_e$, m_e is the mass of the electron, $m_e c^2=0.5\,MeV$.

1) a) Write down the conservation equations.

b) Show that the energy of the π^+ obtained is: $E_{\pi^+}=c^2\dfrac{m_{K^+}^2+m_{\pi^+}^2-m_{\pi^0}^2}{2\,m_{K^+}}$.

c) What is the momentum p_{π^+} of the π^+?

d) Find the numerical values of E_{π^+} in MeV and p_{π^+} in MeV/c.

2) *R'* is the proper frame of π^+. Determine the velocity $\vec{u}$ of this inertial frame with respect to *R*.
3) The charged pion proper mean lifetime is 26 ns, what is the lifetime and the distance traveled in the laboratory?