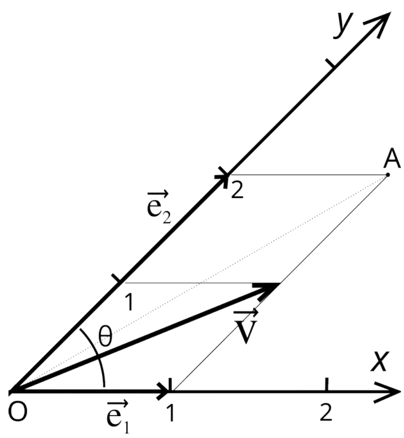
Vector Space, Change of coordinates and Composition of velocities

Exercise 1 Let's consider an euclidean space with a non-normal and non-orthogonal Cartesian basis, with θ=π/4:



a) Give the covariant components g_{ij} of the metric.

b) Write the metric and determine the distance OA.

c) Express the vector $\vec{v}$ with the contravariant components and the covariant basis. Represent the components on the graph.

d) Find the contravariant components g^{ij} of the metric.

e) Determine the contravariant basis and the covariant components of $\vec{v}$. Draw them on the figure.

Exercise 2 Minkowski Diagram

a) On a spacetime diagram, represent the events $E_0(ct=1, x=0)$ and $E_1(ct=\gamma, x=\beta\gamma)$.
E_1 represented for $\beta=1/2$.

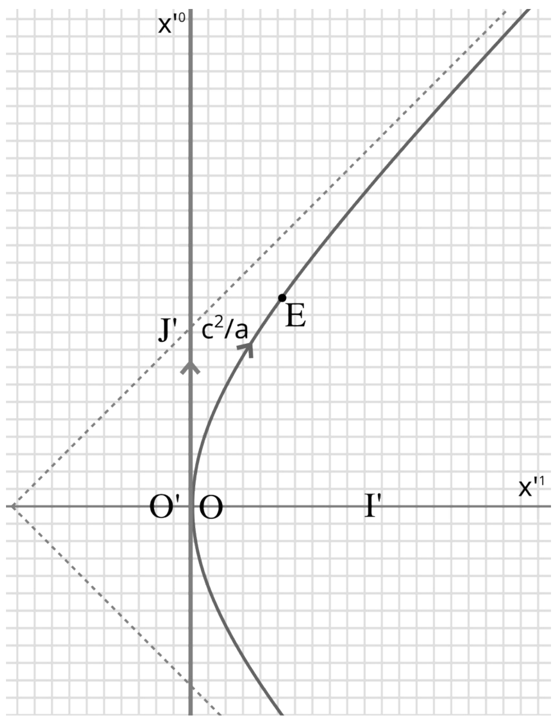
b) Find the pseudonorm of $\widetilde{OE_0}$, $\widetilde{OE_1}$ and $\widetilde{E_O E_1}$. Are they spacelike or timelike vectors?

c) Represent the curve(s) for $OE=\tau=1$.

d) Find an event E_2 as $\widetilde{E_O E_2}$ and $\widetilde{E_O E_1}$ orthogonal and $E_0E_2=1$.

e) From a minkowskian point of view, what kind of triangles are OE_0E_1 and $E_0E_1E_2$?

f) Do we have Minkowskian equilateral triangles? It depends on the definition. Prove there are no triangles with the three sides the same pseudonorm (and then the same kinds, timelike, lightlike or spacelike). However, it is possible if the kinds are mixed.
Example: find a value of β for OE_0E_1 to be an equilateral triangle.



Exercise 3 Metric in an accelerated reference frame. On the Minkowski diagram of the inertial frame R', two worldlines are represented, vertically the one of the observer at rest at O', and on the right the one of an uniformly accelerated rocket in the x direction. R denote the accelerating reference frame. At $t'=0$, $t=0$, and $O'(t'=0)=O(t=0)$. The change of coordinates in given:

$$ct'=\left(x+\frac{c^2}{a}\right)\sinh\left(\frac{a\,t}{c}\right) \quad \text{and} \quad x'=\left(x+\frac{c^2}{a}\right)\cosh\left(\frac{a\,t}{c}\right)-\frac{c^2}{a}$$

Find the metric in the accelerated rocket.

Represent R with the axis (ct, x) at $E=O(t>0)$.

Exercise 4 Give the inverse special Lorentz transform $x^{\mu}(x'^{\nu})$.

Exercise 5

From Earth, we measure with our inertial frame of reference, 5 years after the departure of a spacecraft, a huge stellar eruption produced by the star Alpha Centauri B located 4 light-years away. The ship is moving from Earth to Alpha Centauri with $\gamma=\sqrt{2}$. In the reference frame of the spaceship where and when does the eruption occur?
When the light flare will be seen in a telescope from Earth? From the vessel?
Drawn a space-time diagram to illustrate all the exercise results.

Composition of velocities formula:

Exercise 6

1) Two rockets come face to face with a speed c/2. What is their relative speed?

2) Two rockets travels in the same direction with the speeds 50%c and 75%c.
What is their relative speed?

Exercise 7

Two rockets race in pursuit. Ahead, the pirates' rocket flees at speed 3/4 c. Behind the police rocket speed at c/2 and can't catch up with the pirates. So, from their rocket, the police use a proton cannon, the muzzle velocity of the protons is c/3. Does the protons reach the pirate rocket? Draw also a space diagram to illustrate the situation.

Exercise 8

A laser emits a 650 nm monochromatic red light. An atom absorbs 2.9 eV photons.

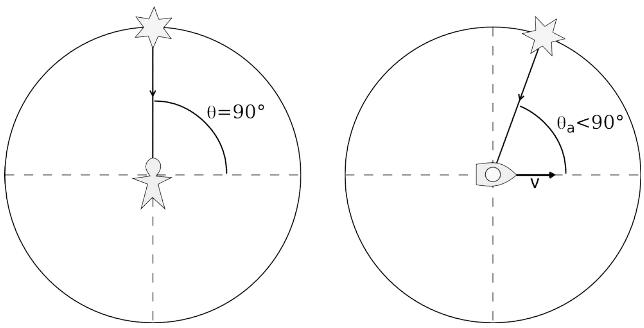
1) As the laser, the atom is at rest and receives the laser light, does it absorb the photons?

2) How the atom has to move to absorb a photon?

Data: $h=6.62\times10^{-34}\,J.s$, $c=3\times10^{8}\,m/s$ *and* $e=1.6\times10^{-19}\,C$.

Exercise 9

Let's determine the change in the perception of the starry sky depending of the speed of the ship. In addition to the change in the perceived color of the stars by *Doppler effect*, their position in the sky is modified, this is called the *aberration of light*. When we are at rest in the galactic frame of reference *R*, the stars are, as a whole, motionless. Let's focus on the case θ=π/2, β=v/c=0.8 and λ=580 nm (yellow).



1) Drawn a Minkowsky diagram (ct, x, y) with $\vec{v}=v\,\vec{i}$, $v>0$, the vessel velocity. Express the equation of the light ray worldline in *R*, and with the help of the Lorentz boost give it in *R'*, the frame of the ship. What's the apparent angle θ_a of the star from the vessel?

2) What's the color of the star from the vessel?